Inertial Forces Posture Control for Humanoid Robots Locomotion

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1. Introduction

In order to evolve in an environment designed for humans, a humanoid robot (or simply humanoid) is supposed capable of performing different motions depending on the given situation. With the walking paradigm in a mature stage, in recent years, the attention of many researchers has passed to more complicated locomotion modes. Some examples are: climbing stairs (Harada et al., 2004), falling down in a less damaging manner (Fujiwara et al., 2004), jumping (Nunez et al. 2005, Sakka et al. 2005) and running (Nagasaka et al., 2004; Kajita et al., 2005), crawling (Kanehiro et al., 2004), etc.

If different control strategies are used for each kind of locomotion, the autonomy and/or versatility of the humanoid can be affected by complicating the problem of implementing each control algorithm and switching between them.

To treat this issue it has been proposed (Tiong & Romyaldy, 2004) to differentiate two important parts on the humanoid locomotion system: the *motion generator* (MG) and the *posture controller* (PoCtr), see Fig. 1. The objective of the former is to define the desired locomotion using some specific parameters and the later will find the angles of all the actuated articulations of the robot such that this motion is achieved.

In this work, we will present a new posture controller which finds the angles of all the articulations of the humanoid (θ) which produces the desired generalized inertial forces ($\mathbf{Q}^{inertial, \text{Ref}}$) and motion of the extremities of the humanoid ($\boldsymbol{\xi}_{i}^{\text{Ref}}$), see Fig 1.

1.1 Description of the approach

The motion generation section describes the input parameters for the PoCtr presented on section 3, and discusses the relationship between the inertial forces and the zmp and angular momentum will be presented.

The inertial force PoCtr is the main result presented in this work. It is based on the Lagrangian equations of motion which relates the accelerations of the actuated articulations with the generalized external forces (force and torque) acting at the support foot of the humanoid. The generalized inertial forces are the derivatives of the linear and angular momentum of the whole humanoid. By controlling these parameters it is possible to consider the *zero moment point* (zmp) stability of the robot.

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Figure 1. Locomotion control system capable of treating different humanoid locomotion modes

The generalized position (position and orientation) ζ_i^{Ref} of the extremities of the robot is achieved by an inverse kinematics algorithm. The implementation of a PD controller provides asymptotic reaching of the hands trajectories, which means that an initial error can be compensated by our controller. The inertial forces $\mathbf{Q}^{inertial,\text{Ref}}$ are retrieved by a computed torque like algorithm.

One of the main objectives of our research is to present algorithms that can be implemented in real humanoid robots. In order to prove the effectiveness of our approach, we tested the method on the humanoid robot HRP-2 (Kaneko et al., 2004), shown on Fig. 2. The robot's motion was symmetric (left and right extremities following the same pattern). This allowed us to consider the robot only on the sagital plane, and to apply our general method in reduced order. The inertial forces were selected to obtain an elliptic trajectory of the CoM and zero angular momentum. This motion was chosen because it is dynamic in the sense that the velocity and acceleration of the CoM are not negligible. The hands motion is to asymptotically reach a circular trajectory after being static at the beginning of the motion.

The method presented in this paper for robots commanded in angle (local PID control for each motor) can be also implemented in torque. On (Nunez et al., 2006) we presented the inertial forces PoCtr considering flight phases and it is applied in torque mode instead of angle mode.

1.2 Related literature

Several works had been presented which covers one or more parts of the block diagram, in Fig. 1, even if this general locomotion paradigm is not often mentioned. Maybe the most representative is the posture controller called Resolved Momentum Control (RMC), presented in (Kajita et al., 2003) and which generalizes the CoM jacobian approach (Sugihara 2003). The inputs for this PoCtr are the desired linear and angular momentum of the whole robot as well as the generalized position of each foot. This posture controller has proven to be useful for treating different locomotion modes like kicking, walking and running (Kajita et al., 2003; Kajita et al., 2005). The advantages of using this kind of methodology are clearly demonstrated on (Sian et al., 2003) where the zmp stability of the motion is considered and the autonomy of the robot is enhanced by the automatic selection of the degrees of freedom (DoF) that should be used for different kind of motions. This approach has been implemented on the humanoid HRP-2 and HRP-2L (Kajita et al., 2005).

The main difference with the work presented here is that RMC is implemented in velocity level (momentum) while the Inertial Forces Posture Controller (IFPC) is implemented in

acceleration level (forces). This opens the possibility of implementing our method in torque and to asymptotically reach the trajectories of the extremities.

The dynamic filters proposed in (Nagasaka et al., 2004) are focused on the motion generation and in particular in the continuity of trajectories of the *Center of Mass* (CoM) for walking, running and jumping motions. This method is implemented on the humanoid QRIO (Nagasaka et al., 2004). Again the locomotion is codified using the generalized motion of the CoM. The posture controller, based on the CoM jacobian, is not detailed on the paper. Regarding the use of whole body dynamics in acceleration level, we can mention the works of (Kondak et al., 2005), based on Tanie's dynamic equations, and (Rok So et al., 2003), based on the Lagrange equations. In both cases the explicit consideration of the zmp constraint instead of a motion generation stage reduces the generality of the approaches.

2. Motion generation

The motion generation stage is supposed to express the desired locomotion (walking, kicking, jumping, etc) using some specific parameters. The generalized inertial forces and position of the extremities are used in our approach, and will be described in this section. We will show how to take into account the zmp stability of the motion using the inertial forces.

2.1 Generalized Position and inertial forces

The generalized position of the extremities (ξ_i) is composed as follows

$$\boldsymbol{\xi}_{i} = \begin{pmatrix} \boldsymbol{r}_{i}^{\mathrm{T}} & \boldsymbol{\gamma}_{i}^{\mathrm{T}} \end{pmatrix}^{\mathrm{T}} \in \boldsymbol{\mathfrak{R}}^{6}$$

where $r_i \in \Re^3$ denotes the cartesian position and $\gamma_i \in \Re^3$ the orientation (Euler angles, for instance) of the frame atached to each extremity *i* which is not in contact with the environment. When the robot is standing on the right foot $i \in \{LF \ RH \ LH\}$, which corresponds to the left foot and right and left hands, as shown on Fig. 2. It is clear that *i* will switch while the robot is walking, running or jumping. In this work we will present in detail the simple suppoer phase. Double support and flight phases are easily obtained implemented based on the same procedure.

The desired generalized position ξ_i^{Ref} of the extremities which are not in contact with the environment should be passed in as inputs to the IFPC presented in next section. The corresponding velocity ξ_i^{Ref} and acceleration ξ_i^{Ref} will be also needed for implementing a PD controller capable of asymptotically tracking these desired trajectories.

One of the key points of our approach is to consider the ground reaction force and moment as acting at a fixed point on the support polygon¹. Let us consider the vertical projection of

¹ The ground reaction force, distributed along the whole foot surface, can be considered as a force with an associated torque. This torque depends on the considered point (Vernon's Theorem). The zmp is only a special point where this torque is zero, but any other point can be considered (Sardain et al., 2004; Goswami et al., 2004).

the right ankle, the point *RH* in Fig. 2, for instance. The generalized reaction force acting at this point is denoted by

$$\mathbf{Q}_{RF} = \begin{pmatrix} \mathbf{f}_{RF}^{\mathrm{T}} & \boldsymbol{\tau}_{RF}^{\mathrm{T}} \end{pmatrix}^{\mathrm{T}} \in \mathfrak{R}^{\mathrm{6}}$$

where f_{RF} and τ_{RF} are the ground reaction force and torque, respectively. The Newton-Euler formulation applied to the whole humanoid shown in Fig. 2 states that

$$\begin{pmatrix} \mathbf{f}_{\mathrm{RF}} \\ \boldsymbol{\tau}_{\mathrm{RF}} \end{pmatrix} + \begin{pmatrix} m_T \mathbf{g} \\ \mathbf{r}_{RF,G} \times m_T \mathbf{g} \end{pmatrix} = \begin{pmatrix} m_T \ddot{\mathbf{r}}_{RF,G} \\ \mathbf{r}_{RF,G} \times m_T \ddot{\mathbf{r}}_{RF,G} + \dot{\mathbf{L}}_G \end{pmatrix}$$
(1)

or

$$\mathbf{Q}_{RF} = \mathbf{Q}_{RF}^{inertial} - \mathbf{Q}_{RF}^{g}$$
(2)

with

$$\mathbf{Q}_{RF}^{inertial} = \begin{pmatrix} m_T \, \mathbf{\ddot{r}}_{RF,G} \\ \mathbf{r}_{RF,G} \times m_T \, \mathbf{\ddot{r}}_{RF,G} + \mathbf{\dot{L}}_G \end{pmatrix}$$
(3)
$$\mathbf{Q}_{RF}^g = \begin{pmatrix} m_T \mathbf{g} \\ \mathbf{r}_{RF,G} \times m_T \mathbf{g} \end{pmatrix}$$

where m_T is the total mass of the robot, $\mathbf{r}_{RF,G}$ is the vector from *RF* to the CoM of the robot, denoted by *G*. The angular momentum of the whole robot around *G* is computed as (Goswami et al., 2004)

$$\mathbf{L}_{G} = \sum_{i} \left(\mathbf{r}_{G,Gi} \times m_{j} \dot{\mathbf{r}}_{G,Gj} + \mathbf{L}_{G}^{j} \right)$$

where for each link j, m_j is the mass, $\mathbf{r}_{G,Gj}$ is the position vector from G to the CoM of the link and $\mathbf{L}_G^j = I^j \boldsymbol{\omega}^j$ is the angular momentum around its own CoM.

As well as the trajectories of the hands and swing foot must be specified in velocity and acceleration, the desired inertial forces must be specified also using the integral like terms:

$$i\mathbf{Q}_{RF}^{inertial} = \begin{pmatrix} m_T \dot{\mathbf{r}}_{RF,G} \\ \mathbf{r}_{RF,G} \times m_T \dot{\mathbf{r}}_{RF,G} + \mathbf{L}_G \end{pmatrix}$$
(4)

$$ii\mathbf{Q}_{RF}^{inertial} = \begin{pmatrix} m_T \mathbf{r}_{RF,G} \\ \int_0^{t} (\mathbf{r}_{RF,G} \times m_T \dot{\mathbf{r}}_{RF,G} + \mathbf{L}_G) d\tau \end{pmatrix}$$
(5)

Notice that the generalized inertial forces expressed in (3) and its integral terms (4) and (5) depend not only on the desired integral of position, velocity and acceleration of the CoM of the robot, but also on the angular momentum around G, its derivative and two integral terms. This means that imposing the desired inertial forces is equivalent to specify the linear and angular momentum of the whole robot. The Integral like terms are required in next section to obtain a closed loop PID control on the inertial forces.



Figure 2. The humanoid robot HRP-2. The position and orientation of the free extremities are denoted by \mathbf{r}_i and γ_i , respectively. The generalized force around a fixed point (*RF*) on the support foot can be decomposed into gravitational and inertial force

2.2 Stable Inertial forces

Considering the generalized ground reaction force as applied at a fixed point of the support polygon implies that the foot must remain flat on the ground during the whole motion. This is because if the foot rolls over an edge or corner the point where the reaction force is supposed to act will not be in contact with the ground.

The foot not rolling over an edge or corner can be characterized using zmp. The contact with flat foot is characterized by $zmp \in S$ where *S* is the support polygon. According to Fig. 2, the zmp can be computed using

$$\tau_{RF} = \tau_{zmp} + \mathbf{r}_{RF,ZMP} \times \mathbf{f}_{RF} \tag{6}$$

where

$$\mathbf{r}_{RF,ZMP} = \begin{pmatrix} x_{RF,ZMP} & y_{RF,ZMP} & 0 \end{pmatrix}^{\mathrm{T}}$$

denotes the vector from RF to the zmp as shown in Fig. 3. By definition the torque τ_{zmp} has only a *z* component, so using eq. (6) we can obtain

$$\tau_{RFx} = y_{RF,zmp} f_z \qquad \tau_{RFy} = -x_{RF,zmp} f_z$$

Considering a square foot, the zmp condition can be stated using the reaction moment as follows

$$-x_{RFB}f_{RF_*} \ge \tau_{RF_*} \ge x_{RFF}f_{RF_*} \tag{7}$$

$$-y_{RFB}f_{RF_{z}} \le \tau_{RF_{y}} \le y_{RFF}f_{RF_{z}}$$

$$\tag{8}$$

where x_{RFB} , x_{RFF} , y_{RFB} and y_{RFF} are used to define the geometry of the foot as shown in Fig. 3.



Figure 3. Forces acting at the support foot and its geomety

The second condition for keeping the support foot on the ground is that the reaction force must be positive

$$f_{RFz} \ge 0 \quad \Rightarrow \quad \ddot{z}_{RF,G} \ge -g \tag{9}$$

This is because the ground can not pull down the foot. The implication comes from (1) with $g = 9.81 m/s^2$. The last necessary condition is to avoid slippage and depends on the friction constant between the foot and the ground, denoted by μ :

$$\frac{\|f_{RF_{z}}\|}{f_{RF_{x}}} \leq \mu \quad \Rightarrow \quad \ddot{x}_{RF,G}\mu \leq \ddot{z}_{RF,G} \tag{10}$$

There are different ways to express the desired motion using the inertial forces. A simple choice is to define a desired trajectory of the center of gravity *G* to obtain \mathbf{f}_{RF}^{Ref} and their integrals. Then, the only remaining term to complete τ_{RF}^{Ref} would be the desired trajectory of the angular momentum around *G*. The choice of this trajectory is not evident because it has an influence on the overall motion of the robot. Making $\mathbf{L}_G = 0$ has been proposed as stability index in (Goswami et al. 2004) and is maybe the first choice to be tried. An important fact is that also a desired zmp can be specified by combining (6) with (1). Finding

the adequate angular momentum or zero moment point for a given motion is still, to the authors best knowledge, an open problem.

The inertial force PoCtr will impose the desired motion which must be coded using the trajectories of hands and swing foot and the desired inertial forces. The proposed motion must be feasible, meaning that the hands reachability and singularity must be considered as well as the stability of the desired locomotion, characterized by equations (7), (8), (9) and (10). This must be done in the motion planning stage and some trial and error is up to now necessary to tune the parameters of the motion like the angular momentum and the zmp trajectory.

3. Inertial force position control

In this section the main contribution of this research is presented. The objective of the inertial force PoCtr is to find the desired angles for all the articulations of the humanoid such that the stable inertial forces and positions described in section 2 are obtained.

In order to obtain the Lagrange equations describing the dynamics of the humanoid robot in ground and aerial motions, it is necessary to consider, besides the vector of internal angles of the robot, denoted by $\theta \in R^n$, the position and orientation of a given point of the robot. Usually the hip generalized position ξ_B is used, but any other point can be considered. For simplicity we will consider the coordinate of the support point, i.e. ξ_{RF} . Notice that the model can be modified simply applying the coordinate change $\xi_{RF} = f(\xi_B, \theta)$. The extended Lagrangian model of the robot with extended coordinates

$$\mathbf{q} = \begin{pmatrix} \boldsymbol{\theta} \\ \boldsymbol{\xi}_{RF} \end{pmatrix} \tag{11}$$

can be written as:

$$D(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{v}\mathbf{C}(\mathbf{q},\dot{\mathbf{q}}) + \mathbf{g}(\mathbf{q}) = \begin{pmatrix} \boldsymbol{\tau} \\ \mathbf{Q}_{RF} \end{pmatrix}$$
(12)

where **vC** is the *vector* of centrifugal and coriolis forces, **g** the gravity forces, τ are the articulation torques, and **Q**_{*RF*} is the generalized force applied at *RF*. Considering (11), it is possible to split (12) for the actuated and non actuated coordinates as

$$D_{11}\theta + D_{12}\ddot{\xi}_{RF} + \mathbf{vC}_1 + \mathbf{g}_1 = \tau$$
(13)

$$D_{21}\ddot{\boldsymbol{\theta}} + D_{22}\ddot{\boldsymbol{\xi}}_{RF} + \mathbf{v}\mathbf{C}_2 + \mathbf{g}_2 = \mathbf{Q}_{RF}$$
(14)

In fact, (13) are *n* equations corresponding to the actuated coordinates while (14) represents 6 equations describing the dynamics of the position and orientation of the support foot. Assuming perfectly inelastic contact, when the robot is standing on its support foot $\xi_{RF} = \xi_{RF} = 0$. Substituting this condition in (14) we can get

$$D_{21}\hat{\theta} + \mathbf{v}\mathbf{C}_2 + \mathbf{g}_2 = \mathbf{Q}_{RF} \tag{15}$$

This equation relates all the actuated angles acceleration to the external force, and this will be the base of our control approach. The reason of using (14) instead of (13) for obtaining our control law is because most of humanoid robots, HRP-2 in particular, are controlled in angles position instead of torques. On (Nunez et al., 2006) the use of (13) for a torque based control law is presented.

3.1 Decoupled position control

Let us separate the actuated angles corresponding to each extremity, i.e. legs and arms, of each end effector as

$$\boldsymbol{\theta} = \begin{pmatrix} \boldsymbol{\theta}_{RF}^{\mathrm{T}} & \boldsymbol{\theta}_{LF}^{\mathrm{T}} & \boldsymbol{\theta}_{LH}^{\mathrm{T}} & \boldsymbol{\theta}_{RH}^{\mathrm{T}} \end{pmatrix}^{\mathrm{T}}$$

where θ_{RF} is the vector containing the angles of the right leg, θ_{LF} the left foot, and the last two are the angles of left and right arms. Supposing that the body is not articulated (or that the articulations are not used), the relative generalized position for each extremity *with respect to the hip* is a function of the relative angles of the corresponding extremity, i.e. ${}^{B}\xi_{i} = \mathbf{f}(\theta_{i})$. Deriving twice this expression we obtain

$${}^{B}\ddot{\xi}_{i} = J_{i}(\theta_{i})\ddot{\theta}_{i} + \dot{J}(\theta_{i},\dot{\theta}_{i})\dot{\theta}_{i}$$

with $i \in \{LF, RH, LH\}$ for the simple support case. If we are dealing with a *standard humanoid robot*, meaning that there are 6 DoF by limb and if furthermore singular configurations are avoided, the jacobian of each extremity J_i is invertible and we can apply the classical PD controller in task space for each extremity as follows

$$\ddot{\boldsymbol{\theta}}^{ref} = J_i^{-1} \left(-\dot{J} \dot{\boldsymbol{\theta}}_i + \mathbf{u} \mathbf{P}_i \right) \tag{16}$$

where the position control input is defined as:

$$\mathbf{u}\mathbf{P}_{i} = {}^{B} \dot{\xi}_{i}^{ref} + kP_{vi} ({}^{B} \dot{\xi}_{i}^{ref} - {}^{B} \dot{\xi}_{i}) + kP_{pi} ({}^{B} \xi_{i}^{ref} - {}^{B} \xi_{i})$$
(17)

where the diagonal matrices kP_{vi} and kP_{pi} must be chosen from stability. The implementation of this control law permits to have initial errors on the desired trajectory of the hands and swing foot, and the rapidity for reaching the target trajectories will depend on kP_{vi} and kP_{pi} .

The over-actuated case can be considered using the pseudo-inverse of the jacobian for optimizing a criteria (manipulability, for instance).

Notice that in order to pass from the absolute position and orientation of the extremities, as mentioned in section 2 to the relatives used in (17) it is necessary to use the hip generalized position ξ_B , because ${}^B\xi_i = \xi_i - \xi_B$. In most humanoid robots, including HRP-2, the position velocity and acceleration of the hip can be obtained from sensors (combining gyroscope, accelerometer signal and their numerical derivatives).

3.2 Inertial force position control

Eq. (15) can be separated as

$$\sum_{i} D_{21i} \ddot{\boldsymbol{\theta}}_{i} + D_{21RF} \ddot{\boldsymbol{\theta}}_{RF} + \mathbf{vC}_{2} + \mathbf{g}_{2} = \mathbf{Q}_{RF}$$
(18)

with $i \in \{LF, RH, LH\}$, considering the simple support phase.

The basic idea of the generalized force position control is to find the angles of the support leg in order to obtain the desired external force and to compensate the desired motion of the upper body given by the decoupled position control.

This idea can be implemented *using whole body dynamics in acceleration level*. In fact substituting expression (2) on eq. (18) we can obtain

$$D_{21RF} \ddot{\boldsymbol{\theta}}_{RF} = \left(\mathbf{Q}_{RF}^{inertial} - \mathbf{v}\mathbf{C}_2 - \sum_i D_{21i} \ddot{\boldsymbol{\theta}}_i \right)$$

The elimination of the gravity term is because the foot coordinates where used as generalized coordinates. It can be shown that \mathbf{g}_2 from (2) is equivalent to $-\mathbf{Q}_{RF}^g$ in (18). Assuming D_{21RF} invertible² our inertia force control takes the following form

$$\ddot{\boldsymbol{\mathcal{B}}}_{RF}^{Ref} = D_{21RF}^{-1} \left(\mathbf{u} \mathbf{F} - \mathbf{v} \mathbf{C}_2 - \sum_i D_{21i} \ddot{\boldsymbol{\mathcal{B}}}_i^{Ref} \right)$$
(19)

with **uF** given as a PID control

$$\mathbf{uF} = \mathbf{Q}_{RF}^{iRef} + kF_{\nu}i\tilde{\mathbf{Q}}_{RF} + kF_{p}ii\tilde{\mathbf{Q}}_{RF} + kF_{i}ii\tilde{\mathbf{Q}}_{RF}$$
(20)

The expressions with $\tilde{}$ are the difference between real and desired values of the inertial forces expressed in eq. (3) and the matrices kF_v , kF_p , kF_i are again chosen following stability and performance criteria.

3.3 Double support phase

During the double support phase (d.s.p.) the decoupled position control (16) with (17) should be implemented for both arms, i.e. $i \in \{LF, RF\}$. Then the force position controller (19) with (20) must be implemented for obtaining the angles of one of the support legs, lets suppose the right one.

The angles of the left leg can be then obtained as follows. The generalized position and velocity of the left foot can be obtained as

$${}^{RF} \xi_{LF} = {}^{RF} \xi_{B} + {}^{B} \xi_{LF}$$
$${}^{RF} \dot{\xi}_{LF} = J_{B,RF}(\theta_{RF})\theta_{RF} + J_{B,LF}(\theta_{LF})\theta_{LF}$$

²It is out of the scope of this paper to consider in detail the invertibility of matrix D_{21RF} . For the time being we can only say that this generalized inertia matrix is invertible in all our simulations and experiments.

Because the left foot is in contact with the ground, $\xi_{LF} = 0$ and the desired angles of the left foot can be obtained simply as

$$\boldsymbol{\theta}_{LF}^{Ref} = -\boldsymbol{J}_{B,LF}^{-1}\boldsymbol{J}_{B,RF}\boldsymbol{\theta}_{RF}^{Ref}$$

In most humanoid robots, including HRP-2, the motor of each actuated articulation is controlled in angular position, meaning that every sampling period the desired position θ^i must be passed as reference. As shown in next section these values can be obtained from angular acceleration or velocities (for the d.s.p) by simple numerical integration.

4. Experiments

In this section, in order to validate the proposed approach, the experiments on the humanoid robot HRP-2 are presented. The selection motion is simple because the objective of the experiments is only to show the applicability of the proposed inertial force posture control.

The selected movement is symmetric with respect to the sagittal plane, meaning that the left and right parts of the robot move identically. As explained in (Nunez et al. 2005) this kind of motions allows us to consider the robot as constrained to the sagittal plane as shown in Fig. 3. In this case the reference motion can be specified using only

$$\mathbf{Q}_{RF}^{i} = \begin{pmatrix} m_{T} \ddot{z}_{RF,G} \\ m_{T} \ddot{x}_{RF,G} \\ m_{T} (z_{RF,G} \ddot{x}_{RF,G} - x_{RF,G} \ddot{z}_{RF,G}) - \dot{L}_{Gy} \end{pmatrix}$$
$$\boldsymbol{\xi}_{RH} = \begin{pmatrix} X_{RH} \\ Z_{RH} \\ P_{RH} \end{pmatrix} \in R^{3}$$

where P_{RH} is the pitch of the right hand. The actuated articulations used were only the elbow, shoulder and hip for the upper body and the ankle, knee and hip for the support leg. This means that $\theta_{RF} \in R^3$ and $\theta_{RH} \in R^3$. For the left hand side the same angles were passed to the corresponding articulations.

Once the desired angular acceleration was obtained using (16), (17), (19) and (20) the desired positions to pass to each articulation were obtained by the simple numerical integration algorithm

$$\theta^{k} = \Delta_{t} \dot{\theta}^{k} + \Delta_{t} \theta^{k-1}$$
$$\theta^{k} = \Delta_{t}^{2} \ddot{\theta}^{k} + \Delta_{t} \dot{\theta}^{k-1} + \theta_{k-1}$$

The same integration method was employed for terms needed in (5) the sample time period being of $\Delta_t = 5ms$. This time was largely enough for making all the computations required by our method.

The desired trajectory of G and the hand position were specified to be ellipsoid as shown in Fig. 4.

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Figure 4. The motion of the robot uses sinus and cosinus functions on the x - y directions in order to obtain the ellipsoid motion shown. During the first 4 *s* the arms of the robot have not relative motion (left) and after that the arms will move to reach the desired trajectory (right). After t = 12s the CoM stay still while the ands continues to move

As mentioned in section 2, the desired trajectory must be specified using first the desired absolute position of the hands ξ^{Ref} and its derivatives. The proposed hand motion was specified using sinus and cosinus functions in the x-z plane with amplitude of 5cm and period of T = 1.5s, this is

$$\boldsymbol{\xi}_{RH}^{\text{Ref}} = \begin{pmatrix} \boldsymbol{x}_{RH} \\ \boldsymbol{z}_{RH} \\ \boldsymbol{\theta} \end{pmatrix} = \begin{pmatrix} 0.05 \sin \left[2\pi (1/1.5)t \right] \\ 0.05 \cos \left[2\pi (1/1.5)t \right] \\ 0 \end{pmatrix}$$

In our experiment in order to verify the closed loop behavior of the controller, we decided to keep the arms fixed during the first 4 seconds of the motion, this means

Hands Motion
$$\begin{cases} t \le 4s \quad \dot{\theta}_{RH} = \ddot{\theta}_{RH} = 0\\ t \ge 4s \quad \xi_{RH}^{\text{Ref}} \end{cases}$$

This means that during this period, only the desired inertial force is specified. We can notice in Fig. 5 and Fig. 6 the transitory after t = 4s before reaching the desired trajectory.



Figure 5. Vertical hands positions. The decoupled position controller is activated at t = 4[s]. Because the proposed control is a PD in closed loop, the desired trajectory is reached after a short transition period



Figure 6 Vertical hands velocity

For the trajectory of the CoM we used cosinus with amplitude of the on the *z* direction of 5[cm] while a sinus of amplitude was 2.5[cm] for the *x* component; both signal with period of T = 2[s]. During the first period the signal grows from 0 to the final amplitude, this is in order to respect the initial condition of the robot being static, that is $\dot{\mathbf{r}}_{RF,G}(t) = \dot{\mathbf{r}}_{RF,G}(t) = 0$. This means

$$\begin{pmatrix} r_{Gx} \\ r_{Gz} \\ f_{L_{Gy}} \end{pmatrix} = \begin{pmatrix} 0.025 \sin\left[2\pi(1/2)t\right] \\ 0.05 \cos\left[2\pi(1/2)t\right] \\ 0 \end{pmatrix}$$
(21)

We can notice that the reference motion of the CoM gradually stops to arrive to zero acceleration at t=12[s]. After this time and before t=18s the IFPC compensate the hands motion in order to keep *G* at a fixed position. This is

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$$CoM \quad Trajectory \begin{cases} t \le 12s & r_G^{\text{Ref}} \\ t \ge 12s & \dot{r}_G^{\text{Ref}} = \dot{r}_G^{\text{Ref}} = 0 \end{cases}$$

The consequent ground reaction force and the signals from the force sensor (located at *RF* in HRP-2) are compared in Fig. 7 and Fig. 8.



Figure 7. Desired and measured ground reaction force on the x direction



Figure 8. Desired and measured ground reaction force on the z direction

Concerning the angular momentum, as shown on (21) we specified $L_{Gy} = 0$ during the whole motion. The consequent ground reaction torque is shown on next figure.

Finally, the desired zmp point, which is consequence of the desired CoM motion and zero angular momentum as reference is shown in Fig. 10.

On Fig. 7 to Fig. 10 we can see that the following of the inertial forces, related to the CoM trajectory, eq. (3), is better when the CoM of the robot is not static, i.e. before t=12s. The difference between real and reference values in those figures may be explained as the effect of the compliance of the contact between the foot and the ground. When the CoM is stopped after t=12s, this phenomenon becomes more important and the robot bounces a little forward and backwards. Because the presented control method supposes rigid contact

between the foot and the ground, this bouncing can not be taken into account. However we can see that the presented controller, besides the noise on force signals, imposes the desired dynamic motion of the CoM and angular momentum in order to obtain the desired stable motion for a given trajectory of the upper body.



Figure 9. Desired and measured ground reaction torque around y axis



Figure 10. With $L_{Gy} = 0$ the distance from *RF* to the *zmp* does not exceed 3.5[*cm*]. As a consequence the foot remains flat on the ground

5. Conclusions and perspectives

In order to consider a whole body control approach capable of treating different kinds of locomotion modes, the consideration of a motion planing and a the posture controller stages is important. In this paper this approach is presented for the locomotion of a humanoid robot. With the proposed approach motions including aerial phases, can be considered. The inertial force posture controller presented here requires the locomotion to be specified using generalized inertial force, besides the trajectory of the extremities not in contact with the ground. This inertial forces can be planed for having zmp stable motion or desired angular momentum.

The angles imposing the robot motion are computed using a decoupled position control and a force controller based on Lagrange equation of motion with external forces applied at one point. The proposed approach was validated on experiments using the humanoid robot HRP-2.

The motion generation for walking and running is our main perspective. Besides, the consideration of the compliance of the ground-foot contact should be considered on future works. Finally the compensation of external forces on the hands (carrying objects) would be an extra for our controller.

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