

# Dynamical Balance of a Humanoid Robot Grasping an Environment

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## Abstract—

This paper shows some preliminary results on the dynamical balance of a humanoid robot grasping an environment. By grasping an environment, it becomes easier for the robot to keep balance. By using the linear programming, a necessary condition for keeping balance of the robot is formulated taking the grasping force into consideration. We show that the occasion exists where the stronger the hand of a humanoid robot grasps the handrail, the larger the region of ZMP for keeping balance becomes. We further show an experimental result of a humanoid robot climbing up a big gap increasing the stability by grasping a handrail.

## I. INTRODUCTION

Since the kinematical structure of a humanoid robot is similar to that of a human, a humanoid robot is expected to realize a variety of motions as a human can do. To realize such motions, a humanoid robot should move not only on the flat plane but also in an unstructured environment. However, it often becomes difficult for the robot to keep balance when it moves in such an unstructured environment.

Fig.1 shows a situation where a humanoid robot climbs up a large gap. Climbing up such a large gap is difficult to realize since it becomes difficult to keep balance of the robot when only one of the feet contacts an environment. However, even in such a case, a humanoid robot can increase the stability by utilizing the interaction between the hand and the environment. As shown in Fig.1, once the hand of a humanoid robot grasps a handrail, it will become easier for the robot to keep balance. We can also expect that, the stronger the hand grasps the handrail, the more stable the robot would become. Based on this consideration, this research aims to give some preliminary answers to the following couple of problems: (1) For a given grasping force of the hand, judge whether or not the robot can keep the dynamical balance, and (2) For a humanoid robot to realize the planned motion stably, how strong the hand has to grasp the handrail.

The ZMP (Zero Moment Point) is defined to be a point on the ground at which the tangential component of the ground reaction moment becomes zero[1]. A humanoid robot can walk on the flat plane with keeping the dynamical balance if the ZMP is included in the convex hull of the foot supporting area. While the ZMP is very commonly

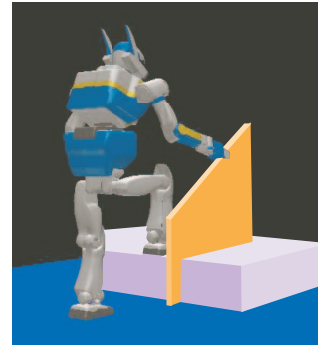


Fig. 1. A humanoid robot climbing up a big gap

used for the gait planning on the flat plane, the number of research on the ZMP allowing the interaction between the hand and environment is limited. Although we sometimes encounter the situation where the hand of a humanoid robot grasps an environment, there has been no research on the ZMP analysis taking the grasping force of the hands into consideration.

In this paper, after showing some previous works, we show the basic idea of the proposed method in Section 3. In Section 4, we formulate the region of ZMP with general 3D frictional contacts by using the linear programming. In Section 5, we confirm the effectiveness of the proposed method by numerical examples. We also show that the method proposed in Sections 3 and 4 is a necessary condition for keeping balance of the robot. In Section 6, we show an extension of the proposed method. Section 7 shows an experimental result.

## II. RELATED WORKS

As for an index of a humanoid robot to keep balance, Vukobratovic et al.[1] introduced the ZMP. The ZMP has been very commonly used for the gait planning of biped/quadruped walking robots such as[9], [10], [11].

For a quadruped robot to walk stably on a sloped surface, McGhee et al.[3] studied the position of the center of gravity for keeping the statical balance of the robot. Yoneda et al.[2] proposed the Tumble Stability Margin focusing on the moment generated by the robot. Papadopoulos et al.[4] considered the maximum inclination angle of the

slope for keeping the dynamical balance of the robot. Messuri et al.[5] and Ghasempoor and Sepehri[6] focused on the potential energy of the robot for keeping the statical and the dynamical balance, respectively. P.B. Wieber[7] proposed the Lyapunov function based method for judging the dynamical balance of the robot.

While the ZMP is commonly used for biped/quadruped robots, the number of research on the ZMP analysis is limited where the supporting points are not limited to the horizontal plane. Kitagawa et al. [8] considered the ZMP for the manipulation tasks and considered ZMP on the sloped surface. Harada et al.[17] studied the ZMP focusing on the 3D convex hull of the supporting points for manipulation tasks.

As for the research on manipulation by a humanoid robot, Harada et al.[12] and Hwang et al.[14] studied the relationship between the external force applied at the hands and the position of ZMP. Yokoyama et al.[13] realized the cooperative work of a humanoid robot with a human.

However, there has been no research on the dynamical balance of a humanoid robot taking the grasping force of the hands into consideration.

### III. BASIC PRINCIPLE

As shown in Fig.2, let us consider the situation where a hand of a humanoid robot grasps a handrail. By modeling the dynamics of the robot using the 2D cart-table model[16], we explain the basic idea of the proposed necessary condition(Fig.2(b)) for keeping balance of the robot. For simplicity, we assume a parallel gripper with a one-dof translational joint as shown in the figure. Also, only in this section, we neglect the effect of friction at each contact point.

When the table tips over, the table will rotate about the edges of the contact area between the table and the floor. However, the finger in the left-side of the hand-rail is fixed to the table, the table does not rotate about the right edge unless the slip occurs at contact area between the table and the floor. Thus, we only consider the situation where the table rotates about the left edge.

Let  $f_{H1}$  ( $\leq 0$ ) and  $f_{H2}$  ( $\geq 0$ ) be the reaction forces applied by the gripper. Also, let  $f_{L1}$  ( $\geq 0$ ) and  $f_{L2}$  ( $\geq 0$ ) be the reaction forces between the table and the floor applied at the edges of the contact area. Here, for the cart-table model,

$$f_{L1} + f_{L2} = mg \quad (1)$$

is satisfied.

We consider the situation where the acceleration of the cart becomes large. In such a case,  $f_{H1} = 0$  is satisfied and the table will rotate about the edge. At the ZMP, since the ground reaction moment becomes zero, we obtain the following equation.

$$0 = -f_{L1}x_Z + f_{L2}(l - x_Z) - f_{H2}z_H, \quad (2)$$

where some comments on this equation are shown in the appendix.

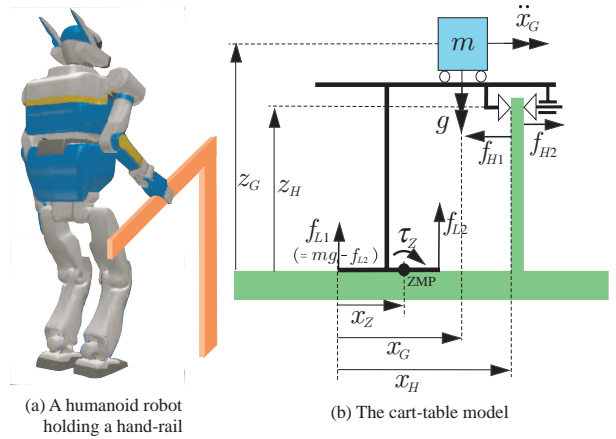


Fig. 2. Explanation of the proposed method

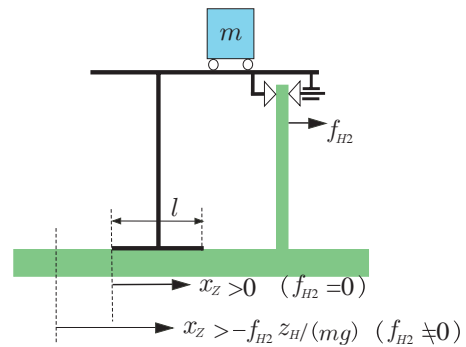


Fig. 3. The region of ZMP of the 2D cart-table model

Solving eq.(2) with respect to the position of the ZMP, the following equation can be obtained:

$$x_Z = (f_{L2}l - f_{H2}z_H)/(mg). \quad (3)$$

Now, let us focus on the region of the ZMP for keeping the dynamical balance of the cart-table system. When both  $f_{L1} > 0$  and  $f_{L2} > 0$  are satisfied, the cart-table model keeps the dynamical balance. From eq.(1), they are equivalent to  $0 < f_{L2} < mg$ . Substituting this relationship into eq.(3), the set of ZMP keeping the dynamical balance can be obtained as:

$$\mathcal{X}_Z = \left\{ x_Z \mid -f_{H2}z_H/(mg) < x_Z, f_{H2} \geq 0 \right\}. \quad (4)$$

This set of ZMP is a function of the grasping force of the gripper as shown in Fig.3. If the grasping force is zero ( $f_{H2} = 0$ ), the set of ZMP is bounded by  $x_Z > 0$  which means that the ZMP exists inside of the contact area between the table and the floor. On the other hand, when the hand is grasping a handrail, the set of ZMP is bounded by  $x_Z > -f_{H2}z_H/(mg)$  which means that the cart-table system can keep the dynamical balance even if the ZMP exists out-side of the contact area. It also means that, if the hand grasps an handrail with stronger grasping force, the robot can keep the dynamical balance more easily. This means that, the stronger the hand grasps a handrail, the more stable the cart-table system will become.

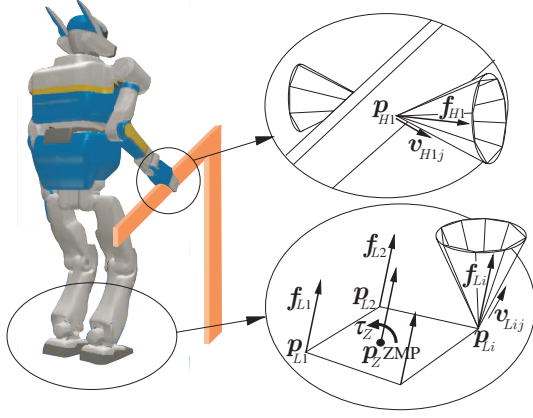


Fig. 4. The contact model

#### IV. 3D FRICTIONAL CONTACTS

In this section, we extend the ZMP analysis to the 3D case assuming frictional contacts at each contact point. As shown in Fig.4, we assume contact points at the vertices of the convex hull of the contact area between the foot and the floor. Let us approximate the friction cone at each contact point by using the  $k$ -faced convex polyhedron. Let  $\mathbf{v}_{Lij}$  be the unit vector along the  $j$ -th edge of the convex polyhedron. Also, let  $\lambda_{Lij}$  ( $j = 1, \dots, k$ ) be the magnitude of contact force along the edge. Since the contact force at each contact point is expressed by the vector sum of the contact forces along the edges of the convex polyhedron, the contact forces can be expressed as

$$\mathbf{f}_{Li} = \mathbf{V}_{Li} \boldsymbol{\lambda}_{Li}, \quad \boldsymbol{\lambda}_{Li} \geq \mathbf{0}, \quad (5)$$

where  $\mathbf{V}_{Li} = [\mathbf{v}_{Lij}, \dots, \mathbf{v}_{Lik}]$ ,  $\boldsymbol{\lambda}_{Li} = [\lambda_{Lij}, \dots, \lambda_{Lik}]^T$ . Also, the contact forces between a finger and the hand-rail are expressed as

$$\mathbf{f}_{Hi} = \mathbf{V}_{Hi} \boldsymbol{\lambda}_{Hi}, \quad \boldsymbol{\lambda}_{Hi} \geq \mathbf{0}, \quad (i = 1, 2) \quad (6)$$

where  $\mathbf{V}_{Hi} = [\mathbf{v}_{Hi1}, \dots, \mathbf{v}_{Hik}]$ ,  $\boldsymbol{\lambda}_{Hi} = [\lambda_{Hi1}, \dots, \lambda_{Hik}]^T$ .

Let  $\mathbf{p}_{Li} = [x_{Li} \ y_{Li} \ z_{Li}]^T$  ( $i = 1, \dots, m$ ) be the position vectors of the vertices of the convex hull of the foot supporting area. Also, let  $\mathbf{p}_{Hi} = [x_{Hi} \ y_{Hi} \ z_{Hi}]^T$  ( $i = 1, 2$ ) and  $\mathbf{p}_Z = [x_Z \ y_Z \ z_Z]$  be the position vectors of the contact points between the hand and the handrail and that of the ZMP, respectively. The ground reaction moment  $\boldsymbol{\tau}_Z$  about the ZMP can be expressed as

$$\boldsymbol{\tau}_P = \sum_{i=1}^m (\mathbf{p}_{Li} - \mathbf{p}_Z) \times \mathbf{f}_{Li} + \sum_{i=1}^n (\mathbf{p}_{Hi} - \mathbf{p}_Z) \times \mathbf{f}_{Hi}, \quad (7)$$

where the horizontal components (first two elements) of the ground reaction moment  $\boldsymbol{\tau}_Z$  about the ZMP are always zero. Substituting eqs.(5) and (6) into eq.(7) and solving with respect to  $x_Z$  and  $y_Z$ , the position of the ZMP can be

expressed as

$$x_Z = \left( \sum_{i=1}^m \mathbf{x}_{Li}^T \boldsymbol{\lambda}_{Li} + \sum_{i=1}^n \mathbf{x}_{Hi}^T \boldsymbol{\lambda}_{Hi} \right) / \left( \sum_{i=1}^m z_{Li}^T \boldsymbol{\lambda}_{Li} + \sum_{i=1}^n z_{Hi}^T \boldsymbol{\lambda}_{Hi} \right), \quad (8)$$

$$y_Z = \left( \sum_{i=1}^m \mathbf{y}_{Li}^T \boldsymbol{\lambda}_{Li} + \sum_{i=1}^n \mathbf{y}_{Hi}^T \boldsymbol{\lambda}_{Hi} \right) / \left( \sum_{i=1}^m z_{Li}^T \boldsymbol{\lambda}_{Li} + \sum_{i=1}^n z_{Hi}^T \boldsymbol{\lambda}_{Hi} \right), \quad (9)$$

$$\boldsymbol{\lambda}_{Li} \geq \mathbf{0}, \quad (i = 1, \dots, m), \quad (10)$$

$$\boldsymbol{\lambda}_{Hi} \geq \mathbf{0}, \quad (i = 1, 2) \quad (11)$$

where  $\mathbf{x}_{Li} = -\mathbf{e}_y^T (\mathbf{p}_{Li} - \mathbf{e}_z z_Z) \times \mathbf{V}_{Li}$ ,  $\mathbf{x}_{Hi} = -\mathbf{e}_y^T (\mathbf{p}_{Hi} - \mathbf{e}_z z_Z) \times \mathbf{V}_{Hi}$ ,  $\mathbf{y}_{Li} = \mathbf{e}_x^T (\mathbf{p}_{Li} - \mathbf{e}_z z_Z) \times \mathbf{V}_{Li}$ ,  $\mathbf{y}_{Hi} = \mathbf{e}_x^T (\mathbf{p}_{Hi} - \mathbf{e}_z z_Z) \times \mathbf{V}_{Hi}$ ,  $\mathbf{z}_{Li} = \mathbf{e}_z^T \mathbf{V}_{Li}$ ,  $\mathbf{z}_{Hi} = \mathbf{e}_z^T \mathbf{V}_{Hi}$ ,  $\mathbf{e}_x = [1 \ 0 \ 0]^T$ ,  $\mathbf{e}_y = [0 \ 1 \ 0]^T$ , and  $\mathbf{e}_z = [0 \ 0 \ 1]^T$ . We note that eqs.(8) ~ (11) show the set of ZMP for keeping the dynamical balance of the robot which corresponds to eq.(4) of the 2D cart-table model.

Next, let us introduce the constraint on the grasping force. The constraints on the grasping forces depend on the joint configuration of the gripper since the joint torque is obtained by multiplying the transpose of the jacobian matrix by the vector of the grasping force. Here, for simplicity, we consider two examples of the joint configuration of the gripper as shown in Fig.5. For the one-dof gripper shown in Fig.5(a), one constraint on the grasping force is imposed. Let us introduce the unit vector  $\mathbf{e}_{n1}$  normal to both the joint axis and the line connecting the center of rotation of the joint and the contact point. The inner product of the grasping force and  $\mathbf{e}_{n1}$  is determined by the joint torque divided by the distance between the joint and the contact point. On the other hand, for the two-dof gripper shown in Fig.5(b), the constraints are imposed on the grasping forces at both of the contact points. For the two-dof gripper, constraint on the grasping force is expressed by using eq.(6):

$$f_{di} = \mathbf{e}_{ni}^T \mathbf{V}_{Hi} \boldsymbol{\lambda}_{Hi}, \quad \boldsymbol{\lambda}_{Hi} \geq \mathbf{0}, \quad (i = 1, 2), \quad (12)$$

where  $\mathbf{e}_{ni}$  denotes the unit vector perpendicular to the plane including both the joint axis and the contact point. While we only focus on simple one or two dof grippers, we can easily extend the formulation to multi-fingered hands unless the grasping forces are underdetermined. For more precise discussion, see[22].

For the cart-table model, the COG moves within the horizontal direction. This condition provides the following constraint on the contact forces.

$$\sum_{i=1}^m z_{Li}^T \boldsymbol{\lambda}_{Li} + \sum_{i=1}^2 z_{Hi}^T \boldsymbol{\lambda}_{Hi} = mg, \quad (13)$$

where  $m$  denotes the total mass of the robot.

Now we obtain the region of the ZMP for keeping the dynamical balance of the robot. The ZMP is defined in

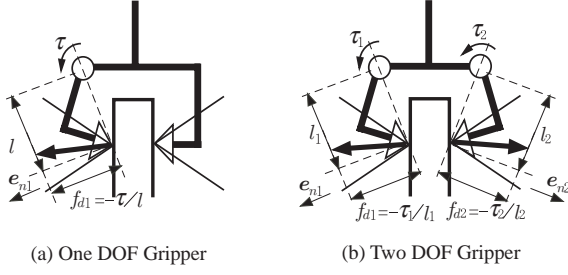


Fig. 5. The constraint on the grasping force depending on Grippers

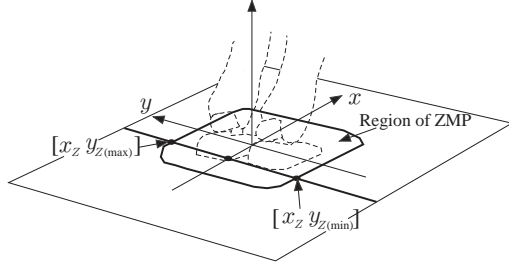


Fig. 6. The explanation of how to solve the LP problem

eqs.(8) and (9). The region of ZMP is limited by eqs. (10), (11), (12), and (13). Although eqs.(8) and (9) are nonlinear equation with respect to  $\tilde{\lambda}_{Li}$  ( $i = 1, \dots, m$ ) and  $\tilde{\lambda}_{Hi}$  ( $i = 1, 2$ ), they can be linearized by using the method shown in the appendix. Now, we formulate the following linear programming problem:

$$\begin{aligned}
 & \text{Minimize/Maximize} && y_Z = \sum_{i=1}^m \mathbf{y}_{Li}^T \tilde{\lambda}_{Li} + \sum_{i=1}^2 \mathbf{y}_{Hi}^T \tilde{\lambda}_{Hi} \\
 & \text{with respect to} && \tilde{\lambda}_{Li} \quad (i = 1, \dots, m), \\
 & && \tilde{\lambda}_{Hi} \quad (i = 1, 2) \\
 & \text{Subject to} && \sum_{i=1}^m \mathbf{x}_{Li}^T \tilde{\lambda}_{Li} + \sum_{i=1}^2 \mathbf{x}_{Hi}^T \tilde{\lambda}_{Hi} = x_Z \\
 & && \sum_{i=1}^m \mathbf{z}_{Li}^T \tilde{\lambda}_{Li} + \sum_{i=1}^2 \mathbf{z}_{Hi}^T \tilde{\lambda}_{Hi} = 1 \\
 & && \mathbf{e}_{ni}^T \mathbf{V}_{Hi} \tilde{\lambda}_{Hi} = f_{di}/(mg) \quad (i = 1, 2) \\
 & && \tilde{\lambda}_{Li} \geq \mathbf{0}, \quad (i = 1, \dots, m) \\
 & && \tilde{\lambda}_{Hi} \geq \mathbf{0}, \quad (i = 1, 2)
 \end{aligned}$$

In the above linear programming problem, the ZMP in the  $x$  direction is one of the constraint conditions, while the ZMP in the  $y$  direction is an objective function. As shown in Fig.6, by solving the linear programming problem for a given  $x_Z$ , we can obtain a point on the boundary of the region of ZMP. By discretizing  $x_Z$  and solving the above linear programming problem for each discretized  $x_Z$ , we can obtain the whole shape of the region of ZMP on the floor.

## V. NUMERICAL EXAMPLE

We confirm the proposed method by numerical examples. The physical parameters of the robot used for the numerical calculation is shown in Fig.7(a). We model the friction cone by a four-faced convex polyhedron, whose

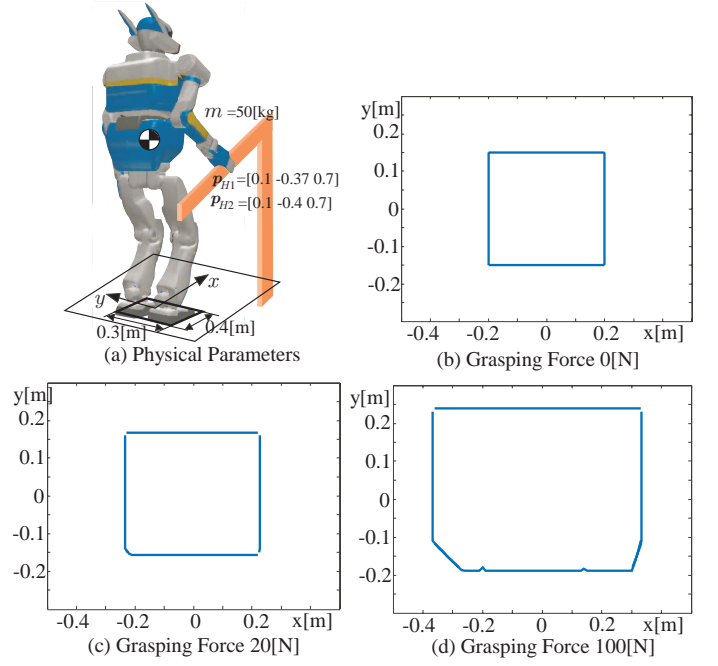


Fig. 7. Simulation Results of ZMP area

friction angle is set to be 30 [deg]. We assume a two dof gripper at the tip of each arm. The region of ZMP for keeping the dynamical balance of the robot is shown in Fig. 7 (b), (c), and (d). As shown in Fig.7(b), the region of ZMP becomes equivalent to the foot supporting area when the grasping force is zero. This is equivalent to the conventional definition of ZMP where the robot can keep the dynamical balance if the ZMP is included in the foot supporting area. On the other hand, the stronger the gripper applies the grasping force, the larger the region of ZMP for keeping balance of the robot becomes. This also indicates that, under a certain condition, the stronger the gripper applies the grasping force, the easier it becomes for the robot to keep balance since the acceleration generated at the COG of the robot can be larger.

In the following, we state why the method proposed in Sections 3 and 4 are necessary condition for keeping balance of the robot. When a humanoid robot stands on a flat floor, the position of the ZMP can be defined as

$$x_Z = \left( \sum_{i=1}^m x_{Hi} f_{zi} \right) / \left( \sum_{i=1}^m f_{zi} \right), \quad f_{zi} \geq 0, \quad (14)$$

$$y_Z = \left( \sum_{i=1}^m y_{Hi} f_{zi} \right) / \left( \sum_{i=1}^m f_{zi} \right), \quad f_{zi} \geq 0, \quad (15)$$

where  $\mathbf{f}_{Hi} \triangleq [f_{xi} \ f_{yi} \ f_{zi}]^T$ . These equations indicate that the ZMP is a convex set and that the robot can keep balance if the ZMP is included in the convex hull of the foot supporting area. In this case, the region of ZMP is independent of the motion of the robot.

On the other hand, when the hand of a humanoid robot contacts an environment, the region of the ZMP changes depending on the motion of the robot[17]. In

the linear programming problem proposed in Section 4, we consider all the possible contact forces within the friction cone constraint (eqs.(10) and the constraint on the grasping force(eq.(12)). For 3D case with friction, the grasping forces are affected by the motion of the robot. In the numerical example shown in Fig.7, (a) shows the necessary and sufficient condition for keeping balance of the robot, while, in (b) and (c), the region of ZMP becomes larger than the actual one. In the next section, we propose a quantitative test for balancing of a humanoid robot grasping an environment taking the motion of the robot into consideration.

## VI. FURTHER EXTENSION

In this section, we will consider extending the proposed method taking the motion of the robot into consideration. Let us assume that the trajectories of the ZMP, the position/velocity of the wrists and the feet, and the joint torque of the hands are given.

Let  $\mathcal{P} = [\mathcal{P}_x \ \mathcal{P}_y \ \mathcal{P}_z]^T$  and  $\mathcal{L} = [\mathcal{L}_x \ \mathcal{L}_y \ \mathcal{L}_z]^T$  be the linear/angular momenta of the robot, respectively. For given trajectories of the ZMP, the linear/angular momenta can be obtained by solving the following differential equations:

$$x_Z = \frac{-\dot{\mathcal{L}}_y + (\dot{\mathcal{P}}_z + Mg)x_G - z_G \dot{\mathcal{P}}_x}{\dot{\mathcal{P}}_z + Mg}, \quad (16)$$

$$y_Z = \frac{\dot{\mathcal{L}}_x + (\dot{\mathcal{P}}_z + Mg)y_G - z_G \dot{\mathcal{P}}_y}{\dot{\mathcal{P}}_z + Mg}, \quad (17)$$

where  $\mathbf{p}_G = [x_G \ y_G \ z_G]^T$  denotes the position of the COG(Center of Gravity) of the robot. The linear/angular momenta are also expressed by the following equations:

$$\dot{\mathcal{P}} = \sum_{i=1}^m \mathbf{V}_{Li} \lambda_{Li} + \sum_{i=1}^n \mathbf{V}_{Hi} \lambda_{Hi}, \quad (18)$$

$$\begin{aligned} \dot{\mathcal{L}} &= \sum_{i=1}^m (\mathbf{p}_{Li} - \mathbf{p}_G) \times \mathbf{V}_{Li} \lambda_{Li} \\ &+ \sum_{i=1}^n (\mathbf{p}_{Hi} - \mathbf{p}_G) \times \mathbf{V}_{Hi} \lambda_{Hi}, \quad (19) \\ \lambda_{Hi} &\geq 0, \quad \lambda_{Li} \geq 0. \end{aligned}$$

Let  $\boldsymbol{\tau} \triangleq [\boldsymbol{\tau}_{L1}^T \ \boldsymbol{\tau}_{L2}^T \ \boldsymbol{\tau}_{A1}^T \ \boldsymbol{\tau}_{H1}^T \ \boldsymbol{\tau}_{A2}^T \ \boldsymbol{\tau}_{H2}^T]^T$  be the joint torque vector where  $\boldsymbol{\tau}_{Li}$ ,  $\boldsymbol{\tau}_{Ai}$ , and  $\boldsymbol{\tau}_{Hi}$  ( $i = 1, 2$ ) denote the torque of the legs, the arms, and the fingers, respectively. The joint torque is obtained as a function of the contact forces as:

$$\boldsymbol{\tau} = \sum_{i=1}^m \mathbf{J}_{Li}^T \mathbf{V}_{Li} \lambda_{Li} + \sum_{i=1}^n \mathbf{J}_{Hi}^T \mathbf{V}_{Hi} \lambda_{Hi} + \mathbf{h}_g, \quad (20)$$

$$\lambda_{Hi} \geq 0, \quad \lambda_{Li} \geq 0,$$

where  $\mathbf{J}_{Hi}$  and  $\mathbf{J}_{Li}$  denote the jacobian matrices of the contact points with respect to the joint variables, and  $\mathbf{h}_g$  shows the effect of gravity force on the joint torque. For given linear/angular momenta of the robot, the joint torque can be calculated based on the inverse dynamics. Thus, we can judge whether or not the robot can keep balance by considering eqs.(18), (19), and (20).

## VII. EXPERIMENT

The experimental environment is shown in Fig.8 where it includes a gap whose height is 28[cm]. Beside the gap, there is a handrail which can be grasped by a humanoid robot.

We performed experiment by using the humanoid robot HRP2[23]. The height, the weight, and the total DOF of HRP2 are  $h = 1.54$ [m],  $m = 58$ [kg], and 30, respectively. The motion of the robot climbing up a large gap is shown in Fig.9. In this experiment, the motion of the robot was relatively slow. In the figure, the robot grasps the hand rail(Fig.9(a)~(c)), steps the left foot onto the gap(Fig.9(d)), shifts the position of the COG(Fig.9(e)), steps the right foot onto the gap(Fig.9(f),(g)), and stands up by releasing the hand from the handrail(Fig.9(h)). When the robot shifts the position of the COG(Fig.9(e)), it is impossible to keep balance without using the interaction between the hand and the environment. Also, when stepping the right foot onto the gap(Fig.9(f),(g)), it becomes difficult for the robot to keep balance without grasping the handrail. The experimental result is shown in Fig.10. The robot can stably climb up the gap by grasping the handrail.



(a) Overview of experimental setup



(b) Step, handrail, and pushed object

(c) Table

Fig. 8. Experimental environment

## VIII. CONCLUSIONS

In this paper, we analyzed the motion of a humanoid robot grasping a handrail. We formulated the region of ZMP for keeping the dynamical balance of the robot by using the linear programming. We showed the occasion where the robot becomes more stable if the the environment is grasped more strongly. We also performed an experiment of climbing up a large gap by grasping the handrail.

The formulation of the ZMP analysis taking the motion of the robot is considered to be our future research topic. Also, when performing an experiment of grasping a

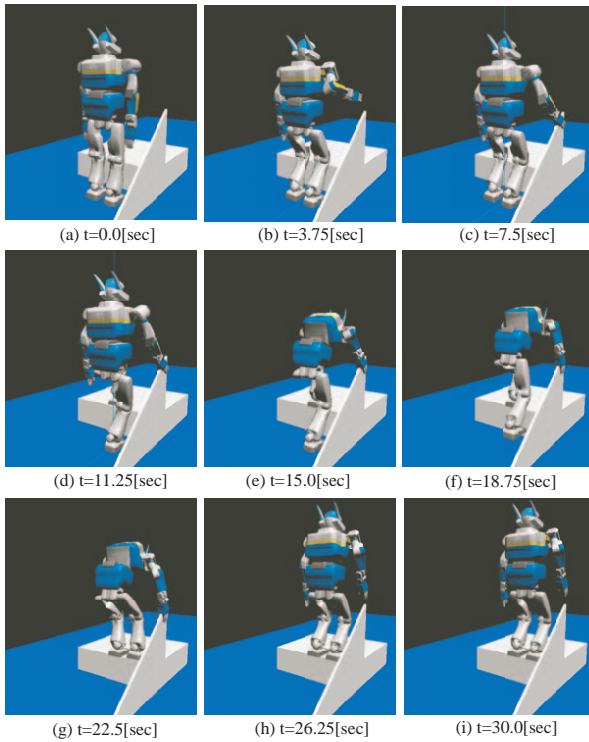


Fig. 9. Overview of planned motion

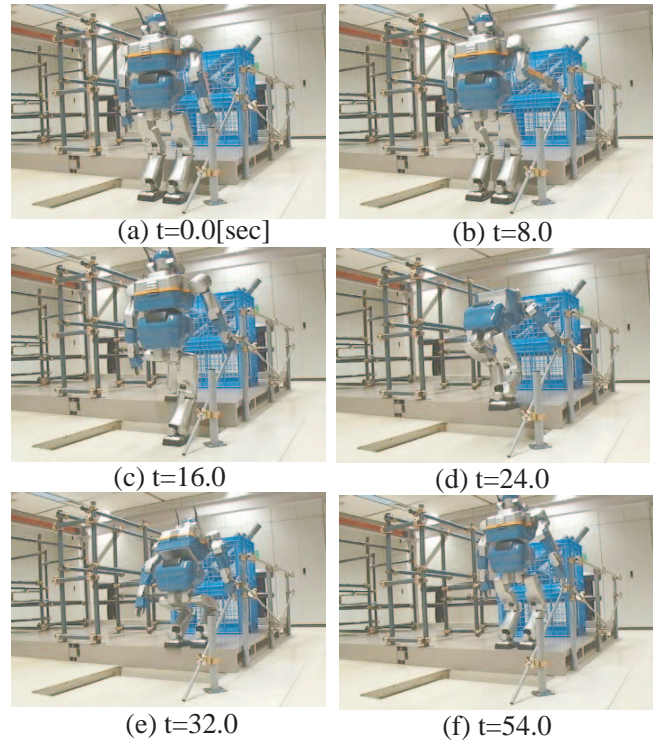


Fig. 10. Experimental Result

handrail, the motion of the robot was slow enough almost satisfying the statical balance of forces. We will plan the motion of the robot grasping an environment, and realize the dynamical motion of climbing up the gap with grasping a handrail. Furthermore, since the experiment was performed without using the vision sensors, the measurement of the position of the handrail by using the vision sensor is also considered to be our future research topic.

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## APPENDIX

### Additional Comments on eq.(1)

There are two kinds of ZMPs when the hand contacts an environment[17]. In the ZMP studied in this paper, we consider all forces acting on the robot as sources of the ground reaction moment. The ground reaction moment  $\tau_Z$  is given by

$$\tau_Z = -f_{L1}x_Z + f_{L2}(l - x_Z) - f_{H2}z_H, \quad (21)$$

$$= mg(x_G - x_Z) - m\ddot{x}_G z_G. \quad (22)$$

We note that the second line of the above equation is same as that of a humanoid robot whose hands do not contact an environment. Solving above equations, the position of ZMP is

$$x_Z = (f_{L2}l - f_{H2}z_H)/(mg), \quad (23)$$

$$= x_G - \frac{\ddot{x}_G}{g}z_G. \quad (24)$$

This ZMP is same as the Generalized ZMP(GZMP) studied in [17].

On the other hand, when we only consider the forces acting at the contact between the floor and the foot, the ground reaction moment  $\tau'_Z$  is given by

$$\tau'_Z = -f_{L1}x'_Z + f_{L2}(l - x'_Z), \quad (25)$$

$$= mg(x_G - x'_Z) - m\ddot{x}_G z_G + f_{H2}z_H, \quad (26)$$

where the first line of the above equation is same as that of a humanoid robot whose hands do not contact an environment. Solving these equations, we can also obtain the position of ZMP. This ZMP can be measured by using force/torque sensor at the ankle of the robot.

### Transformation to LP problem

Let us consider the following nonlinear programming problem:

$$\begin{aligned} \text{Minimize or Maximize} \quad & z = \mathbf{a}^T \mathbf{x} / \mathbf{b}^T \mathbf{x} \\ \text{With respect to} \quad & \mathbf{x} \\ \text{Subject to} \quad & \mathbf{A}\mathbf{x} = \mathbf{c} \\ & \mathbf{x} \geq \mathbf{0} \end{aligned}$$

The solution of this nonlinear programming problem is equivalent to that of the following linear programming problem when  $\mathbf{b}^T \mathbf{x} > 0$  [20], [21]:

$$\begin{aligned} \text{Minimize or Maximize} \quad & z = \mathbf{a}^T \tilde{\mathbf{x}} \\ \text{With respect to} \quad & \tilde{\mathbf{x}}, t \\ \text{Subject to} \quad & \mathbf{A}\tilde{\mathbf{x}} = \mathbf{c}t \\ & \mathbf{b}^T \tilde{\mathbf{x}} = 1 \\ & \tilde{\mathbf{x}} \geq \mathbf{0}, t > 0 \end{aligned}$$