Department of Mathematics, University of Toronto MAT 137Y, 2007-2008 Winter Session	Problem Set #14 Deadline: Thursday April 3, 6:10 p.m. Assignment Posted/Revised: March 28, 2008, 16:26

Please be reminded of the following rules:

Required Information. The front page must include your name, student number, your tutorial code (which will be assigned to you when tutorial rooms are announced), and the name of your teaching assistant. *Failure to put your name and/or your student number will result in a zero in your assignment. Failure to put the name of your TA or your tutorial code will result in a 20% reduction of your assignment mark.* A cover page is not required as long as the necessary information is on the top of the first page.

Paper Size and Requirements. Assignments must be submitted on letter-sized (8.5 $\times$ 11 inch) paper. *Using ripped notebook paper is unacceptable and will result in a zero in your assignment mark.* Assignments that are more than one page in length must be stapled in the top left corner. *Failure to staple such assignments will result in a 20% reduction of your assignment mark.* Do not use clear plastic binders.

Submitting your assignment. You must hand your assignment to your instructor before the beginning of lecture, or deposit the assignment into the MAT 137Y Assignment Box located inside SS 1071. *The penalty for late assignments is zero for the assignment, regardless of the excuse. Assignments handed in after 6:10 p.m. on Thursday will not be accepted for any reason, even if it is one minute late!*

Policy on Plagiarism on Assignments. It is very helpful to have other students with whom to study, and we encourage you to work together. However, **it is extremely important that problem set solutions be written up independently, otherwise this constitutes plagiarism! Don't copy other people's work, and don't let others copy your work!** The teaching assistants will enforce this rule very strictly, and will apply severe penalties to any one in violation. In particular, the Department of Mathematics reminds all students that plagiarism, cheating, and all forms of academic misconduct will not be tolerated. Students in violation of the *Code of Student Conduct* will be dealt with severely by the Department of Mathematics and the Faculty of Arts & Science.

Supplementary Problems. "SHE" refers to the textbook by Salas, Hille, and Etgen (10th Edition)

1. SHE 11.2: 11, 21, 29, 35, 45, 51, 55, 59.
2. SHE 11.3: 5, 13, 23, 25, 29, 39, 45, 53, 57.
3. SHE 12.1: 5, 9, 17, 21, 23.
4. SHE 12.2: 3, 7, 9, 21, 27.
5. SHE 12.3: 3, 7, 9, 13, 17, 21, 25, 31, 33, 37.
6. SHE 12.4: 1, 5, 11, 17, 19, 23, 27, 29, 33.

Required Problems. Hand in solutions to all the problems below.

1. SHE 11.2: 8, 20, 26.
2. SHE 11.3: 10, 26, 30.
3. Consider the sequence defined recursively by $a_1 = 2$ and $a_{n+1} = \dfrac{1}{3 - a_n}$ for $n \geq 1$.
   (a) Show that $0 < a_n \leq 2$ for all positive integers n.
   (b) Show that the sequence $\{a_n\}$ is decreasing.
   (c) Apply a theorem to show that the sequence converges and find the limit.

4. Consider the points on the square $P_1 = (0,1)$, $P_2 = (1,1)$, $P_3 = (1,0)$, and $P_4 = (0,0)$. We construct further points as follows: P_5 is the midpoint of P_1P_2, P_6 is the midpoint of P_2P_3, P_7 is the midpoint of P_3P_4, and so on. This creates a polygonal spiral path $P_1P_2P_3P_4P_5P_6P_7\ldots$.

   (a) Draw a diagram of the path $P_1P_2P_3\ldots P_{15}$ to see that the path eventually approaches a point which we denote P.

   (b) Let the point $P_n = (x_n, y_n)$. Find a recursive relationship for x_n, and show that $\frac{1}{2}x_n + x_{n+1} + x_{n+2} + x_{n+3} = 2$ for all positive integers n.

   (c) Find similar recursive and non-recursive formulas for y_n, and prove your non-recursive formula is true for all positive integers n.

   (d) What are the coordinates of P? (You may assume that the sequences $\{x_n\}$ and $\{y_n\}$ converge by part (a).)

5. SHE 11.4: 8, 16, 26.

6. SHE 12.1: 10, 18.

7. SHE 12.2: 6, 14, 16, 30.

8. SHE 12.3: 4, 12, 16, 20.

9. SHE 12.4: 2, 6, 26.

10. Suppose $\sum a_k$ and $\sum b_k$ are both convergent series with positive terms. Does it follow that $\sum a_k b_k$ converges? Show your answer is correct by an appropriate proof or counterexample.

11. Define a sequence $\{a_k\}$ as follows:

$$a_1 = 1; \qquad a_{k+1} = \frac{2+\cos k}{\sqrt{k}} a_k, \quad k \geq 1.$$

    Does $\sum a_k$ converge or diverge?