Department of Mathematics, University of Toronto MAT 137Y, 2007-2008 Winter Session	**Problem Set #13** **Deadline: Thursday March 27, 6:10 p.m.** Assignment Posted/Revised: March 19, 2008, 15:45

Please be reminded of the following rules:

Required Information. The front page must include your name, student number, your tutorial code (which will be assigned to you when tutorial rooms are announced), and the name of your teaching assistant. *Failure to put your name and/or your student number will result in a zero in your assignment. Failure to put the name of your TA or your tutorial code will result in a 20% reduction of your assignment mark.* A cover page is not required as long as the necessary information is on the top of the first page.

Paper Size and Requirements. Assignments must be submitted on letter-sized (8.5×11 inch) paper. *Using ripped notebook paper is unacceptable and will result in a zero in your assignment mark.* Assignments that are more than one page in length must be stapled in the top left corner. *Failure to staple such assignments will result in a 20% reduction of your assignment mark.* Do not use clear plastic binders.

Submitting your assignment. You must hand your assignment to your instructor before the beginning of lecture, or deposit the assignment into the MAT 137Y Assignment Box located inside SS 1071. *The penalty for late assignments is zero for the assignment, regardless of the excuse. Assignments handed in after 6:10 p.m. on Thursday will not be accepted for any reason, even if it is one minute late!*

Policy on Plagiarism on Assignments. It is very helpful to have other students with whom to study, and we encourage you to work together. However, **it is extremely important that problem set solutions be written up independently, otherwise this constitutes plagiarism! Don't copy other people's work, and don't let others copy your work!** The teaching assistants will enforce this rule very strictly, and will apply severe penalties to any one in violation. In particular, the Department of Mathematics reminds all students that plagiarism, cheating, and all forms of academic misconduct will not be tolerated. Students in violation of the *Code of Student Conduct* will be dealt with severely by the Department of Mathematics and the Faculty of Arts & Science.

Supplementary Problems. "SHE" refers to the textbook by Salas, Hille, and Etgen (10th Edition)

1. SHE 11.7: 3, 5, 15, 19, 27, 37, 51, 55.
2. SHE 12.6: 4, 8, 14, 16.

Required Problems. Hand in solutions to all the problems below.

1. SHE 11.7: 10, 26, 54.
2. (a) Prove for all positive integers n that $\lim_{x\to 0^+} x(\ln x)^n = 0$.

   (b) Prove for all positive integers n that $\int_0^1 (\ln x)^n \, dx = (-1)^n n!$.
3. SHE 12.6: 4, 8, 14, 16.

   Recall that $R_n(x)$, generally known as the remainder term or error term, is defined such that $R_n(x) = f(x) - P_n(x)$, or $f(x) = P_n(x) + R_n(x)$. In lecture we discovered that if $|f^{(n+1)}(x)| \le M$ for all $x \in (-d,d)$, then

   $$|R_n(x)| \le \frac{M|x|^{n+1}}{(n+1)!} \qquad \text{for all } x \in (-d,d).$$

   For all remaining exercises, use the above formula when using the remainder formula or finding error terms.
4. SHE 12.6: 22, 24.

5. For certain functions where the n-th derivative is difficult to compute, it is still possible to find their Taylor polynomials as long as they are either simple products or compositions.

   Consider $\sin x = P_{2n+1}(x) + R_{2n+1}(x)$, where $P_{2n+1}(x)$ is the $2n+1$-st Taylor polynomial of $\sin x$.

   (a) Show that $P_{2n+1}(x^2)$ is the $4n+2$-nd Taylor polynomial of $\sin(x^2)$ by showing that $\lim_{n\to\infty} R_{2n+1}(x^2) = 0$.

   (b) Find the $4n+5$-th Taylor polynomial of $g(x) = x^2 + x^3 \sin(x^2)$.

   (c) Find $g^{(2007)}(0)$ and $g^{(2008)}(0)$.

6. Use Taylor polynomials to approximate the following numbers with error less than ε.

   (i) $\sin \frac{1}{2}$, $\varepsilon = 10^{-3}$.

   (ii) $e^{-1/2}$, $\varepsilon = 10^{-5}$.

   (iii) $\ln \frac{11}{10}$, $\varepsilon = 10^{-3}$.

7. (a) Suppose $P_n(x)$ is the n-th Taylor polynomial for $f(x)$ and $|f^{(n+1)}(x)| \leq M$ for all $x \in (-d,d)$. Prove that $\lim_{x\to 0} \dfrac{R_n(x)}{x^n} = 0$.

   (b) Using the result from part (a), solve the following limits.

   (i) $\lim_{x\to 0} \dfrac{e^x - 1 - x}{x^2}$.

   (ii) $\lim_{x\to 0} \dfrac{1 + x + \frac{x^2}{2} - e^x}{\sin x - x}$.

   Do not use L'Hôpital's Rule to perform either limit.