

Please be reminded of the following rules:

Required Information. The front page must include your name, student number, your tutorial code (which will be assigned to you when tutorial rooms are announced), and the name of your teaching assistant. *Failure to put your name and/or your student number will result in a zero in your assignment. Failure to put the name of your TA or your tutorial code will result in a 20% reduction of your assignment mark.* A cover page is not required as long as the necessary information is on the top of the first page.

Paper Size and Requirements. Assignments must be submitted on letter-sized (8.5×11 inch) paper. *Using ripped notebook paper is unacceptable and will result in a zero in your assignment mark.* Assignments that are more than one page in length must be stapled in the top left corner. *Failure to staple such assignments will result in a 20% reduction of your assignment mark.* Do not use clear plastic binders.

Submitting your assignment. You must hand your assignment to your instructor before the beginning of lecture, or deposit the assignment into the MAT 137Y Assignment Box located inside SS 1071. *The penalty for late assignments is zero for the assignment, regardless of the excuse. Assignments handed in after 6:10 p.m. on Thursday will not be accepted for any reason, even if it is one minute late!*

Policy on Plagiarism on Assignments. It is very helpful to have other students with whom to study, and we encourage you to work together. However, **it is extremely important that problem set solutions be written up independently, otherwise this constitutes plagiarism! Don't copy other people's work, and don't let others copy your work!** The teaching assistants will enforce this rule very strictly, and will apply severe penalties to any one in violation. In particular, the Department of Mathematics reminds all students that plagiarism, cheating, and all forms of academic misconduct will not be tolerated. Students in violation of the *Code of Student Conduct* will be dealt with severely by the Department of Mathematics and the Faculty of Arts & Science.

Supplementary Problems. "SHE" refers to the textbook by Salas, Hille, and Etgen (10th Edition)

1. SHE 4.3: 7, 9, 33, 35, 41.
2. SHE 4.4: 3, 7, 13, 21, 37, 43, 45.
3. SHE 4.5: 3, 7, 9, 15, 23, 25, 31.
4. SHE 4.6: 5, 11, 17, 41, 45, 47.
5. SHE 4.7: 5, 9, 13, 19, 23, 25, 33.
6. SHE 4.8: 3, 21, 27, 43, 47.
7. SHE 4.12: 1, 3, 5, 13.
8. SHE 11.5: 1, 5, 9, 13, 21, 27, 45.
9. SHE 11.6: 1, 5, 7, 15, 31.

Required Problems. Hand in solutions to all the problems below.

1. For the following functions on their respective intervals, determine the points where the local and absolute extrema occur.
 - (i) $f(x) = (x^2 - 1)^3$; $[-1, 2]$
 - (ii) $f(x) = x - 2\cos x$; $[0, \pi/3]$

2. SHE 4.3: 38.
3. SHE 4.4: 36.
4. SHE 4.5: 14, 44.
5. A rain gutter is constructed from a metal sheet 30 cm wide by bending up one-third of the sheet on each side through an angle θ . How should θ be chosen so that the gutter will carry a maximum amount of water?
6. Find the point on the parabola $y = 1 - x^2$ at which the tangent line cuts from the first quadrant the triangle with the smallest area.
7. SHE 4.6: 10, 48.
8. For each of the following limit statements, give the formal definition and prove the statement directly using the formal definition.

(i) $\lim_{x \rightarrow \infty} \sqrt{x} = \infty$.

(ii) $\lim_{x \rightarrow 2^-} \frac{1}{(x-2)^3} = -\infty$.

Warning: If you have been reading the textbook, you will notice that the authors use informal notation to denote limits to infinity and limits at infinity. For example, the textbook often writes “ $\frac{\sin x}{x} \rightarrow 0$ as $x \rightarrow \infty$ ”, or “ $\frac{1}{3-x} \rightarrow -\infty$ as $x \rightarrow 3^+$ ”. We ask all students to avoid this notation and to use standard notation when stating infinite limits, for example,

$$\lim_{x \rightarrow \infty} \frac{\sin x}{x} = 0, \quad \lim_{x \rightarrow 3^+} \frac{1}{3-x} = -\infty.$$

9. Evaluate the following limits.

(i) $\lim_{x \rightarrow \infty} \frac{x^4 + 3x^2 + 1}{x^2(3x+1)(x-3)}$.

(ii) $\lim_{x \rightarrow -\infty} (x + \sqrt{x^2 + 5x})$.

10. SHE 4.7: 8, 22, 30.

11. For the following functions, do a full analysis of the function itself, the first derivative, and the second derivative. Provide information about the domain, intercepts, and (vertical, horizontal, and slant) asymptotes. Find and classify all critical points, and find all intervals where the function is increasing or decreasing. Find all intervals of concavity, and inflection points. Finally, sketch the graph of the function, using the information you obtained.

(0) $f(x) = \frac{x}{1+x^2}$

Note: Do not hand part (0) in. This question will be done in tutorial upon request. To simplify matters a bit, we give the equations for the derivatives:

$$f'(x) = \frac{1-x^2}{(1+x^2)^2}, \quad f''(x) = \frac{2x(x^2-3)}{(1+x^2)^3}.$$

- (i) SHE 4.8: 12
- (ii) SHE 4.8: 36
- (iii) SHE 4.8: 42

A few comments are in order. Because of the length of this question, we will guarantee that one of these functions above will be marked by your TA, and will be worth the equivalent of *two* problems.

Also, do *not* use graph paper when drawing graphs. You should make sure that your graph is big enough so that it clearly illustrates the properties that you have found in your analysis.

12. SHE 4.12: 6, 14(a). For question 6 you may use a calculator.

13. Evaluate the following limits.

(i) $\lim_{x \rightarrow 0} \frac{1 - \cos x}{x^2 + x}.$

(ii) $\lim_{x \rightarrow 0} \frac{\tan x - x - \frac{1}{3}x^3}{x^5}.$

(iii) $\lim_{x \rightarrow 0} \left(\frac{4}{x^2} - \frac{2}{1 - \cos x} \right).$

14. Let $f(x) = \begin{cases} \frac{\sin x}{x}, & x \neq 0, \\ 1, & x = 0. \end{cases}$

(a) Find $f'(0)$.

(b) Find $f''(0)$.

Note: This is the last assignment due before the second term test. The term test will cover all material from the beginning of the course, with an emphasis on Problem Sets 6 through 8. More information regarding the term test will be posted on the course website.